

Feedforward Control

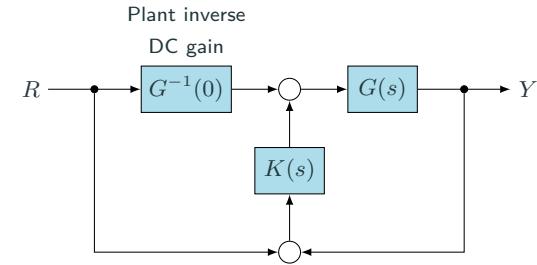
Goal Zero steady-state error

Solution Add an integrator to 'learn' the required steady-state input for zero steady-state error

Problem Integrators typically decrease damping / stability of the system

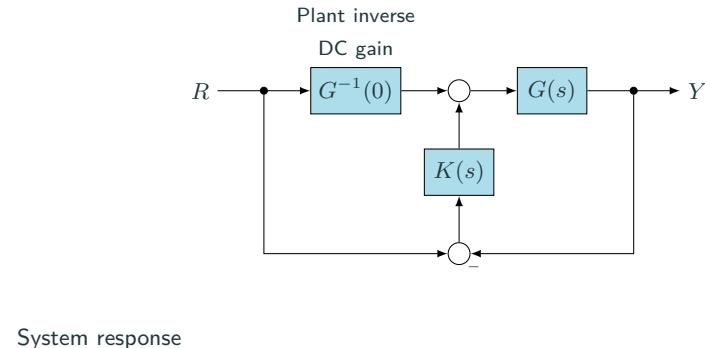
Idea Compute the required steady-state input and add it to the system

Feedforward Control



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Feedforward Control - Analysis



$$\begin{aligned}
 Y &= G(s)(G^{-1}(0)R + K(s)(R - Y)) \\
 (1 + G(s)K(s))Y &= G(s)G^{-1}(0)R + G(s)K(s)R \\
 Y &= \frac{G(s)G^{-1}(0) + G(s)K(s)}{1 + G(s)K(s)}R
 \end{aligned}$$

Feedforward Control - Analysis

System response

$$Y = \frac{G(s)G^{-1}(0) + G(s)K(s)}{1 + G(s)K(s)}R$$

Steady-state output in response to a step input $R = \frac{1}{s}$

$$\begin{aligned}
 \lim_{t \rightarrow \infty} y(t) &= \lim_{s \rightarrow 0} sY(s) \\
 &= \lim_{s \rightarrow 0} s \frac{G(s)G^{-1}(0) + G(s)K(s)}{1 + G(s)K(s)} \frac{1}{s} \\
 &= \frac{G(0)G^{-1}(0) + G(0)K(0)}{1 + G(0)K(0)} \\
 &= 1
 \end{aligned}$$

If we can estimate the steady-state gain of the system, $G(0)$, then we can achieve zero steady-state error without the destabilizing influence of the integrator.

Example

Design a Feedforward Controller

Goal: Overshoot around 20% and zero steady-state error

$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

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$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

Design proportional controller $K(s) = K_p$ to give approximately 20% error $\rightarrow \zeta \approx 0.45$

$$\begin{aligned} \frac{Y}{R} &= \frac{GK_p}{1 + GK_p} \\ &= \frac{K_p}{s^2 + 1.4s + 1 + K_p} \end{aligned}$$

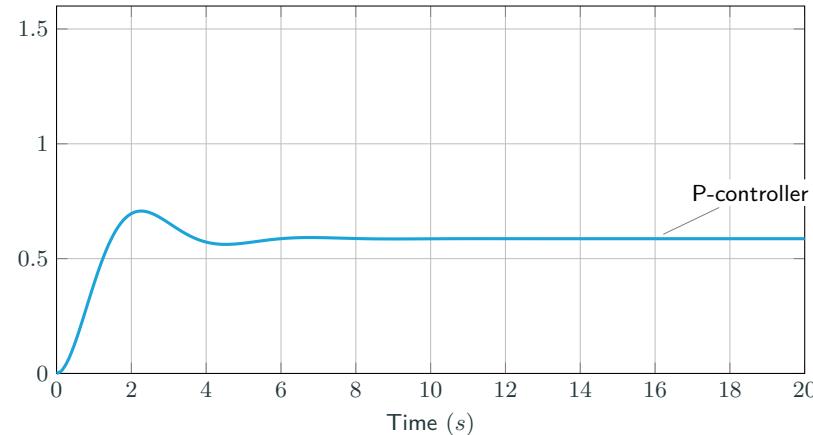
$$\omega_n^2 = 1 + K_p$$

$$2\zeta\omega_n = 1.4 \quad \rightarrow \quad \zeta = 0.45 = \frac{0.7}{\sqrt{1 + K_p}} \quad \rightarrow \quad K_p = 1.4$$

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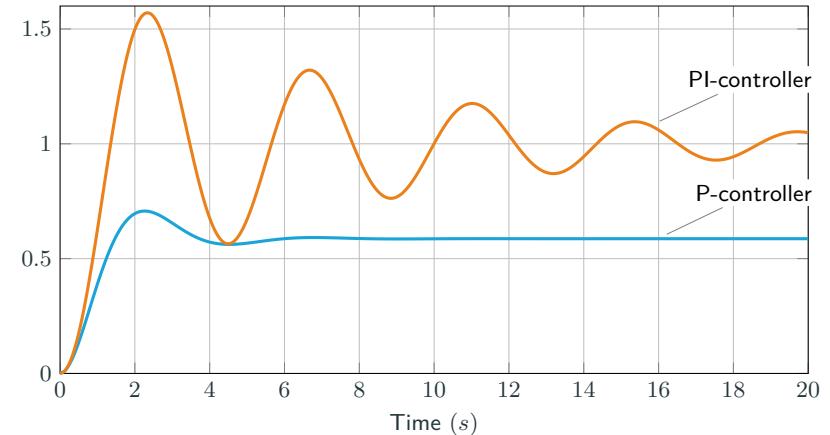
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Example - Closed-Loop Step Response



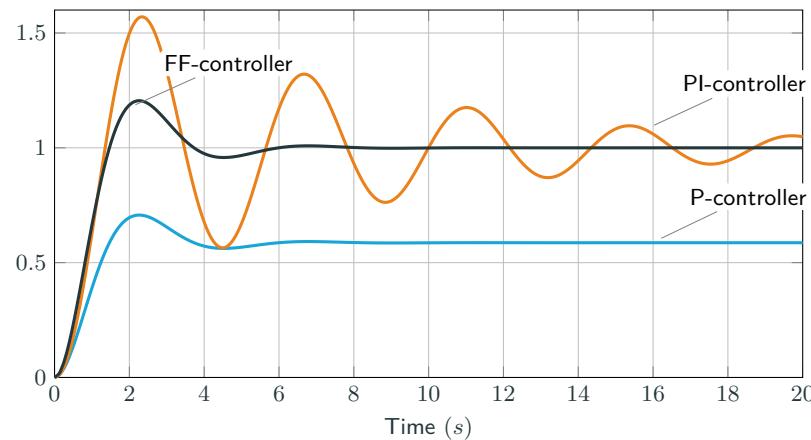
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Example - Closed-Loop Step Response



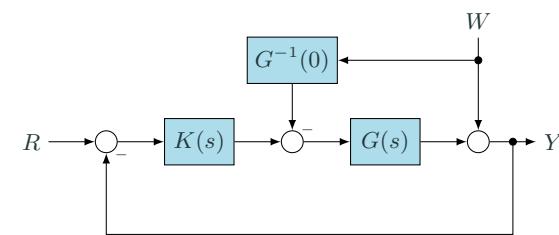
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Example - Closed-Loop Step Response



Feedforward Disturbance Rejection

If a constant disturbance can be estimated or measured, then it can also be mitigated via feedforward control



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Summary - Feedforward Control

Idea Compute the required steady-state input to the system, and add this to the input

- When possible, this removes the need for an integrator, which can reduce the damping of the system
- If possible to estimate the steady-state impact of a disturbance, this can also be removed

Much more advanced feedforward controllers are possible

- Compute the required input for a desired **dynamic** trajectory
- Drive the system with this pre-computed trajectory, and then use feedback only to deal with errors
- Possible to handle some very complex non-linear and constrained system in this way

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